Electromagnetic radiation from free electrons

Contents

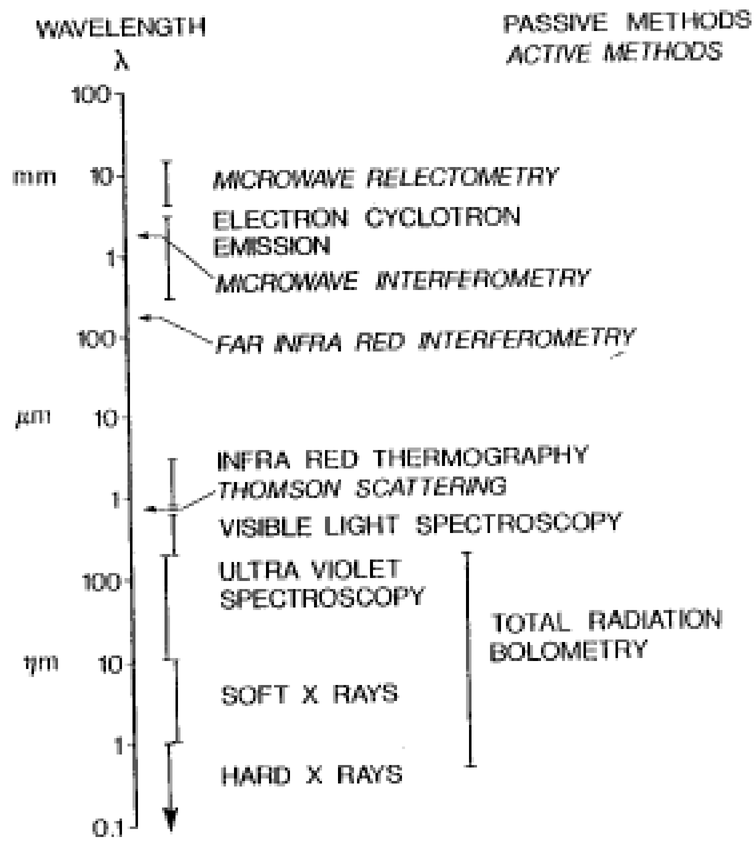
Cyclotron radiation

- **Theory**
- **Electron cyclotron motion**
- **Propagation of perpendicular waves**
- **Typical detection systems**
  - lens
  - waveguide system
  - optical system
- **Results from TEXT**

e-m radiation detectors

- **General considerations**
- **Types**
- **Construction**
- **Heterodyne detection**

Cerenkov radiation



Photon emmission wavelengths from a typical plasma

Cyclotron Radiation

Theory

Calculate the following:

1. radiation from an accelerated charge
   the frequency spectrum in the far field, radiated energy per unit solid angle per angular frequency

2. specific case of a single electron in a B field. The sin, cos terms are expressed as sums of Bessel functions i.e. we will get harmonics

3. sum over a distribution function to get the emissivity

4. approximate to non-relativistic case, and a Maxwellian. Find Doppler broadened Gaussian line widths.

Note that we have to consider radiation absorption. So, change tack!

5. note that in thermodynamic equilibrium a black body emits with a unique intensity, so that the intensity emitted is just the black body level, and gives a measure of temperature

6. worry about polarization and density effects

Electron cyclotron motion

Consider charged particle in a uniform B field. Take B in z direction, so that

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \boldsymbol{B}$$

$$m\dot{\mathrm{v}}_{\mathrm{x}} = qB\mathrm{v}_y \quad m\dot{\mathrm{v}}_{\mathrm{y}} = -qB\mathrm{v}_x \quad m\dot{\mathrm{v}}_{\mathrm{z}} = 0$$

$$\ddot{\mathrm{v}}_{\mathrm{x}} = \frac{qB}{m}\dot{\mathrm{v}}_y = -\left(\frac{qB}{m}\right)^2 \mathrm{v}_x$$

$$\ddot{\mathrm{v}}_{\mathrm{y}} = -\frac{qB}{m}\dot{\mathrm{v}}_x = -\left(\frac{qB}{m}\right)^2 \mathrm{v}_y$$

i.e. a simple harmonic oscillator with a frequency

$$\omega_c = \frac{|q|B}{m}$$

$$\mathrm{v}_{x,y} = \mathrm{v}\ \exp\left(\pm i\omega_c t + i\delta_{x,y}\right)$$

± depends on charge q. Choose phase so that $\mathrm{v}_{\mathrm{x}} = \mathrm{v}\ \exp\left(i\omega_c t\right) = \dot{x}$, and v is positive (speed in plane perpendicular to B). Then

$$\mathrm{v}_{\mathrm{y}} = \frac{m}{qB}\dot{\mathrm{v}}_x = \pm\frac{1}{\omega_c}\dot{\mathrm{v}}_x = \pm i\mathrm{v}\ e^{i\omega_c t} = \dot{y}$$

Integrate once:

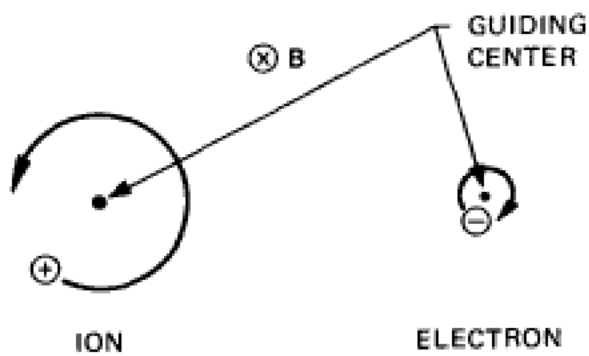
$$\mathrm{x} - \mathrm{x}_0 = -i\frac{\mathrm{v}}{\omega_c}e^{i\omega_c t} \quad \mathrm{y} - \mathrm{y}_0 = \pm i\frac{\mathrm{v}}{\omega_c}e^{i\omega_c t}$$

Larmor radius is

$$r_L = \frac{v}{\omega_c} = \frac{mv}{|qB|}$$

Then real part gives

$$x - x_0 = r_L \sin(\omega_c t) \quad y - y_0 = \pm r_L \cos(\omega_c t)$$



Note: can derive classical transport coefficients now! Electron cyclotron emmission occurs at harmonics of the electron cyclotron frequency:

$$\omega = n\omega_c = n\frac{|q|B}{m}$$

Suppose that the field is known, so that

$$B = B_0 f(R)$$

Then there is a simple transformation between frequency and space

$$\omega = n\frac{|q|B_0 f(R)}{m}$$

e.g. in a tokamak

$$\omega = n\frac{|q|B_0 R_0}{mR}$$

with R the major radial coordinate now. The emission is localized in space, so spatially resolved information is possible from looking at the spectrum. The radiation frequency is decreased for relativistic electrons by $1/\gamma$, and for radiation traveling at an angle θ wrt the B field there is a Doppler shift caused by the relative motion in a parallel direction between the emitting electron and the observer: with $\gamma = \left(1-\beta^2\right)^{-1/2}$; $\beta = v/c$:

$$\omega = n\frac{|q|B_0R_0}{mR}\frac{1}{\gamma}\frac{1}{\left(1-\beta_{||}\cos(\theta)\right)}$$

It is a general principle that if a body emits radiation then it can absorb it at the same frequency. Absorption coefficient $\alpha(\omega)$ is defined as the fractional rate of absorption of radiation per unit path length. $j(\omega)$ is the emissivity. In a medium with refractive index = 1, the radiation intensity $I(\omega)$, the radiated power per unit area per unit solid angle per unit angular frequency, is governed by

$$\frac{dI(\omega)}{ds} = j(\omega) - I\alpha(\omega)$$

with ds along path s. Integrate to obtain

$$I(s_2) = I(s_1)e^{(\tau_1-\tau_2)} + \int_{s_1}^{s_2} j(\omega)e^{(\tau-\tau_2)}ds$$

and the optical depth is

$$\tau = \int^{s}\alpha(\omega)ds$$

e.g. take j/α to be constant, and consider a slab

$$I(s_2) = I(s_1)e^{(-\tau_{21})} + \int_{\tau_1}^{\tau_2}(j/\alpha)e^{(\tau-\tau_2)}d\tau$$
$$= I(s_1)e^{(-\tau_{21})} + (j/\alpha)\left[1-e^{-\tau_{21}}\right]$$

where $\tau_{21} = \tau_2 - \tau_1$ is the total optical depth of the slab. i.e. the emerging intensity is a fraction of the incident intensity plus an additional emitted intensity. For $\tau_{21} >> 1$, optically thick, we have

$$I(s_2) = (j/\alpha)$$

and the slab absorbs all the radiation at the specified frequency incident upon it.

Now a body in thermodynamic equilibrium that is black (i.e. absorbs all radiation) emits radiation with a unique intensity (Planck's radiation law)

$$I(\omega) = B(\omega) = \frac{\hbar\omega^3}{8\pi^3 c^2}\frac{1}{\left(e^{\hbar\omega/T}-1\right)}$$

For low frequencies $\hbar\omega << T$ we have the Rayleigh Jeans form:

$$I(\omega) = \frac{\omega^2 T}{8\pi^3 c^2}$$

Measuring the emission in a line (i.e. at a given ω) leads to T. Because ω depends on space ($\omega = n|q|B_0 R_0/(mR)$ in a toroidal device) we have T as a function of space. Note the emerging intensity in the optically thick limit is the black body radiation (Kirchoff's law).

$$B(\omega) = \frac{j}{\alpha}$$

This allows us to deduce the absorption coefficient immediately from the emissivity, or vice versa..

Propagation of perpendicular waves

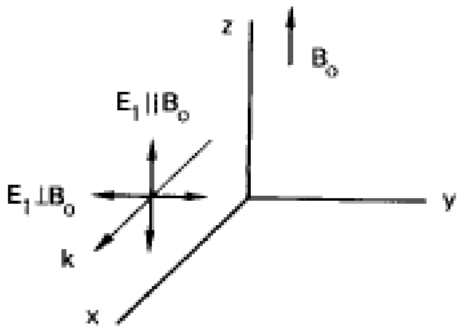
Two characteristic waves: O mode (wave electric field ***E*** parallel to equilibrium magnetic field ***B***) and extraordinary mode (wave ***E*** perpendicular to equilibrium ***B***). For O mode need $\omega > \omega_p$ for propagation. For E mode need $\omega_H < \omega < \omega_r$ or $\omega < \omega_L$, where

$$\omega_R = \frac{\omega_c}{2}\left[1 + \sqrt{1 + 4\frac{\omega_p^2}{\Omega^2}}\right]$$

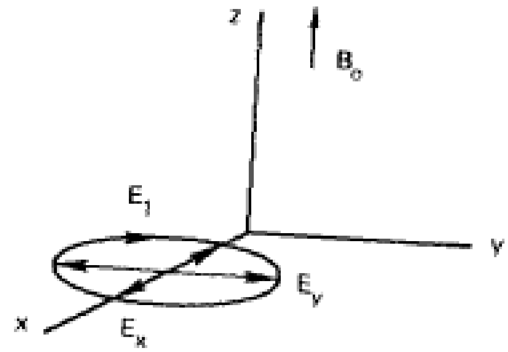
$$\omega_L = \frac{\omega_c}{2}\left[-1 + \sqrt{1 + 4\frac{\omega_p^2}{\Omega^2}}\right]$$

$$\omega_H^2 = \omega_c^2 + \omega_p^2$$

ω_c is just the cyclotron frequency. So for propagation we must have $\omega > \omega_p$. In high density plasmas where $\omega_p > \omega_c$ the lowest harmonics will not be usable.



Geometry for electromagnetic waves propagating at right angles to B_0.



The E-vector of an extraordinary wave is elliptically polarized. The components E_x and E_y oscillate 90° out of phase, so that the total electric field vector E_1 has a tip that moves in an ellipse once in each wave period.

Spatially varying B fields. Consider case where B field varies slowly. For a specific frequency ω_0, the cyclotron emission and absorption at the mth harmonic is appreciable only in a narrow band and

$$m\omega_c(s) - \omega_0 << \omega_0$$

Now the total optical depth through the resonance layer at frequency ω_0 is (assuming gradient of $m\omega_c$ is constant across the layer)

$$\tau_m = \int \alpha_m(\omega_0) ds = \int \alpha_m(\omega_0) \left| \frac{d(m\omega_c)}{ds} \right|^{-1} d(m\omega_c)$$

$$= \alpha_m(s) \left| \frac{d(\omega_c)}{ds} \right|^{-1} \int \varphi(\omega_0 - m\omega_c) d(\omega_c)$$

α_m in the final expression is the frequency integrated absorption coefficient and φ is the line structure constant,

$$j_m(\omega) = j_m \varphi(\omega - \omega_m); \quad \int \varphi(\omega) d\omega = 1$$

φ is narrow so that

$$\int \varphi(\omega_0 - m\omega_c) d\omega_c = \frac{1}{m} \int \varphi(\omega_0 - m\omega_c) d\omega_0 = \frac{1}{m}$$

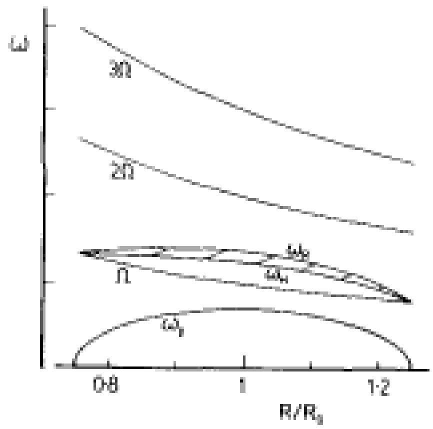
Then

$$\tau_m = \frac{\alpha_m(s)}{m\left|\frac{d(\omega_c)}{ds}\right|} = \frac{L\alpha_m(s)}{m\omega_c}$$

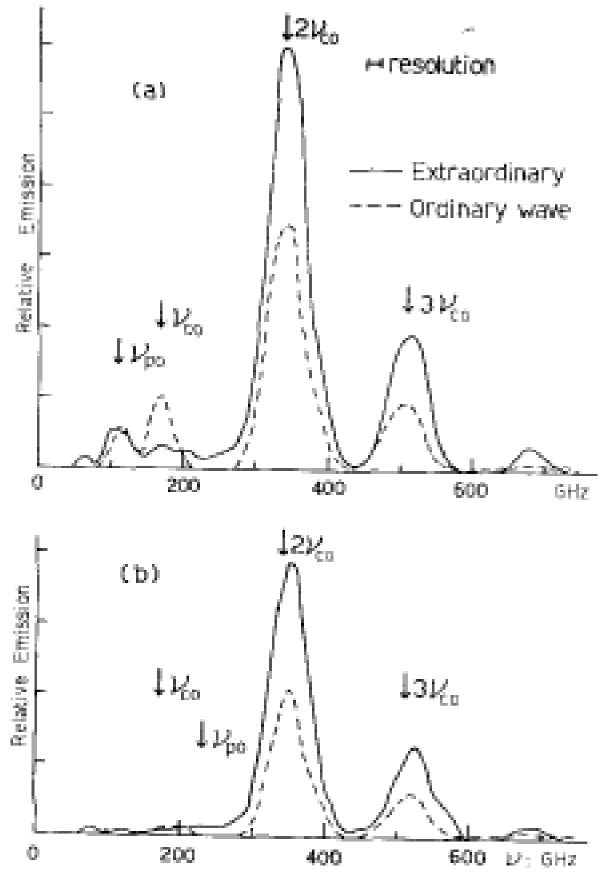
The plasma parameters used to calculate $\alpha_m(s)$ are those at the resonance where $\omega_0 = m\omega_c(s)$. If the plasma is viewed from vacuum then the intensity observed will be

$$I(\omega_0) = \frac{\omega_0^2 T(s)}{8\pi^3 c^2}\left(1 - e^{-\tau_m}\right)$$

as long as the frequency ω_0 is resonant only at one position.



Typical frequency dependencies in a toroidal device



Typical spectra

Typical detection systems

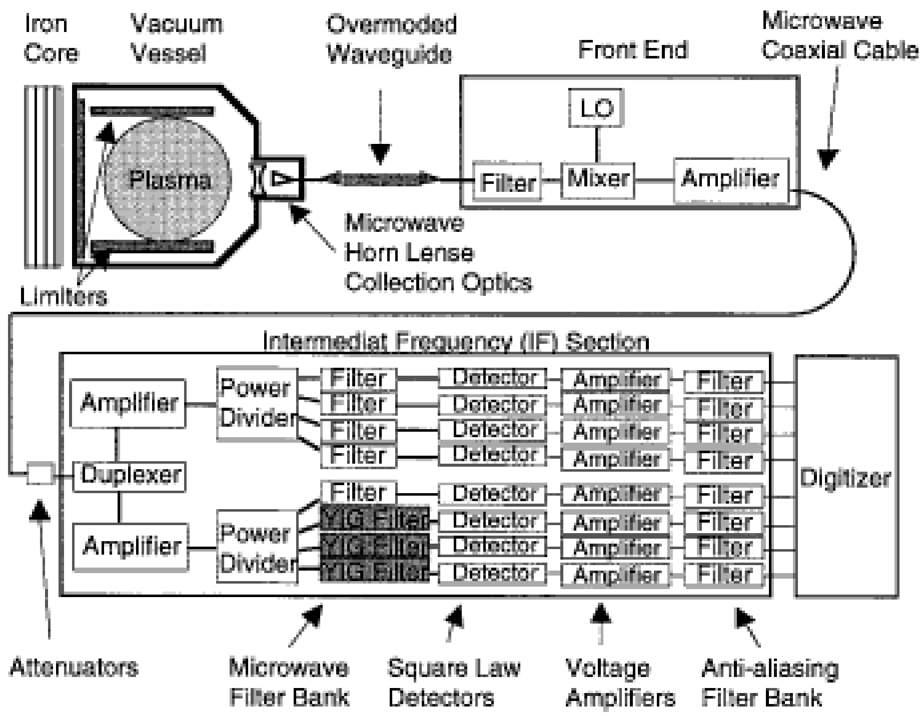
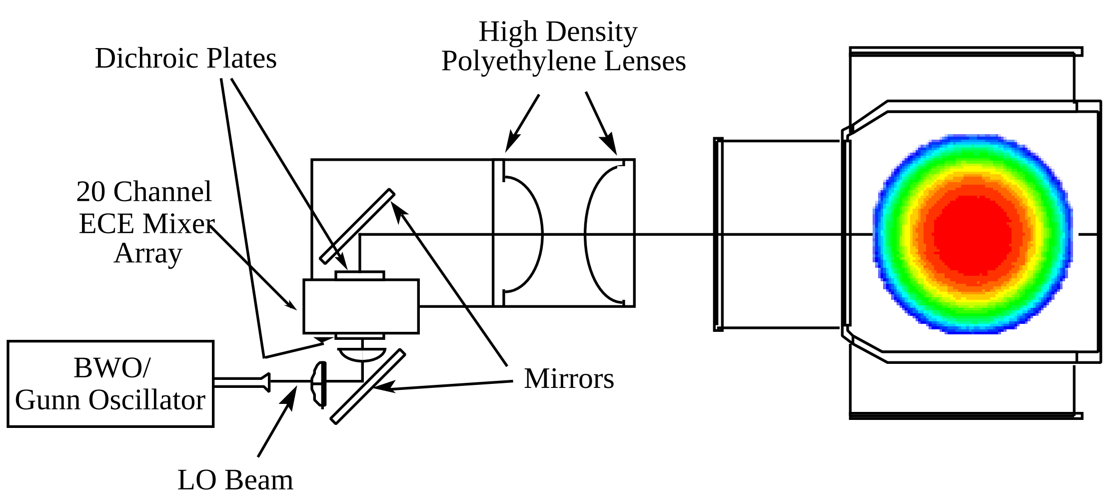
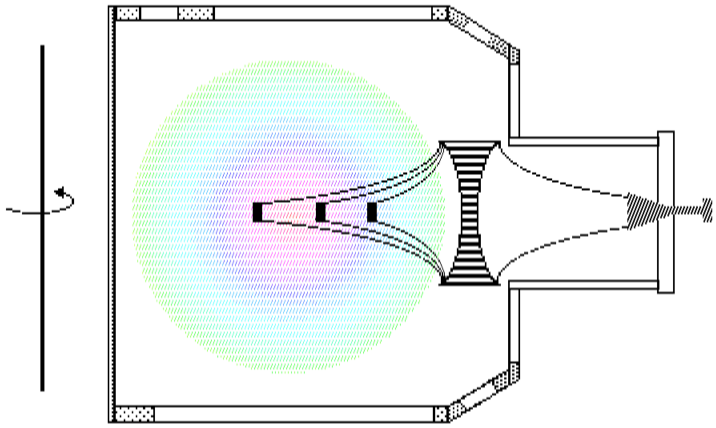
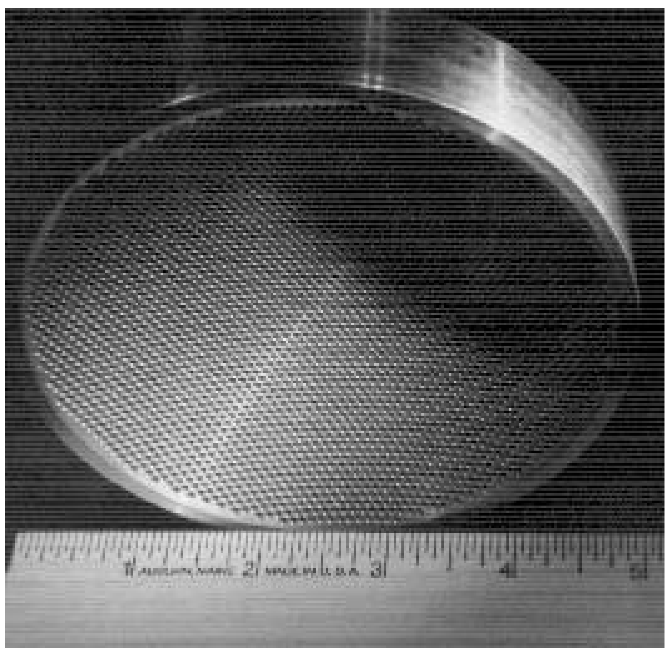


Figure 5.1: Collection optics and detection system for the TEXT-Heterodyne system.



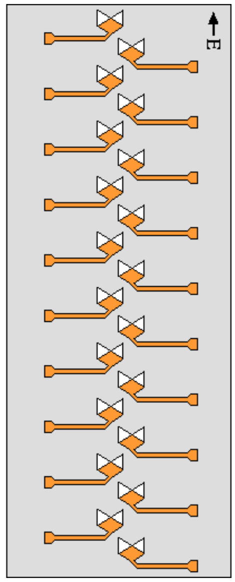
Lens: either polyethylene or metal. Metal lens: smaller 'tubes' increase phase velocity, i.e. modify refractive index, i.e. we have a lens.





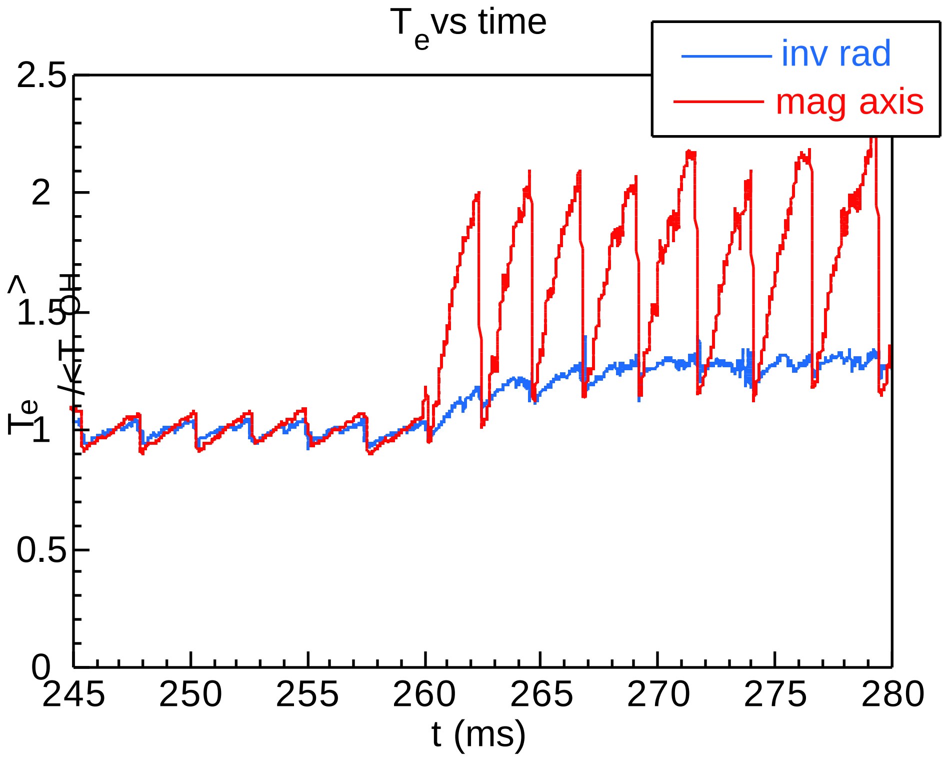
Waveguide system: waveguide selects wavenumber k of cyclotron radiation: allows us to minimize viewed sample volume size. The waveguide also filters against unwanted (lower) frequencies. The local oscillator frequency and signal are mixed on a diode in a cavity at the end of the waveguide. (LO is fed in with a directional coupler).

Optical system: all k's collected and focused onto an antennae followed by a diode. The local oscillator is also focused on to the antennae. A dichroich plate (many parallel waveguides, with a selected cutoff frequency determined by the hole size) filters unwanted lower frequencies.



As in a radio, the heterodyne detection downshifts the frequency from ≈ 100 GHz to ≈ 10 GHz. From now on all components are cheaper!

Results from TEXT



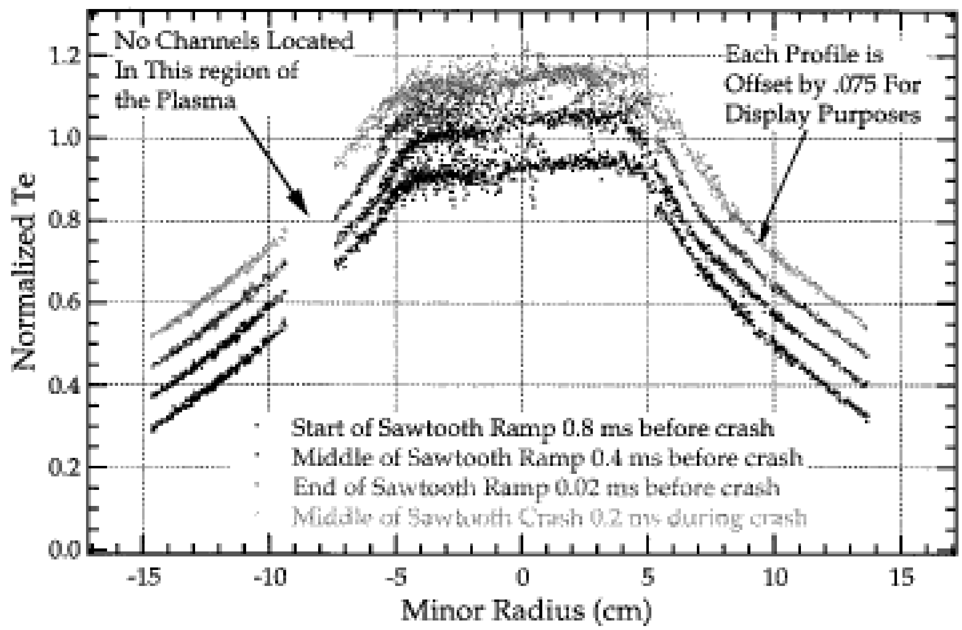


Figure 8.10:Reconstruction of an electron temperature profile for a quiescent discharge in TEXT-U for various phases of the sawtooth cycle.

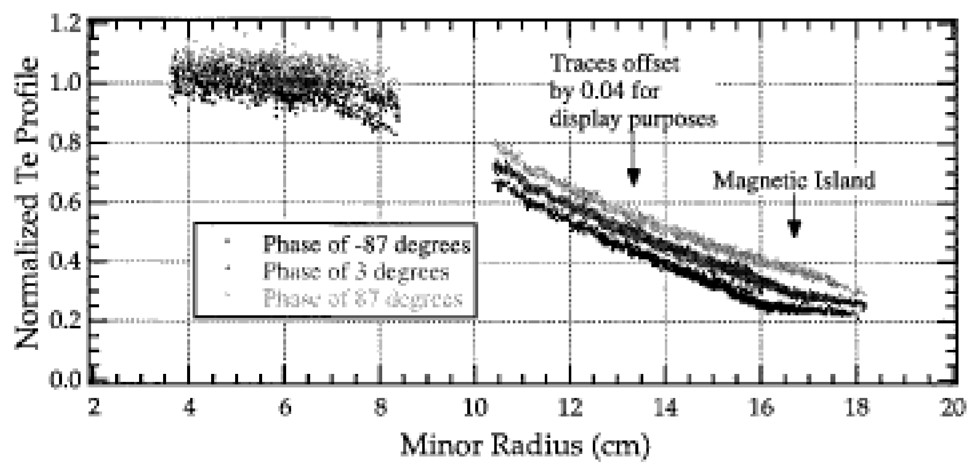
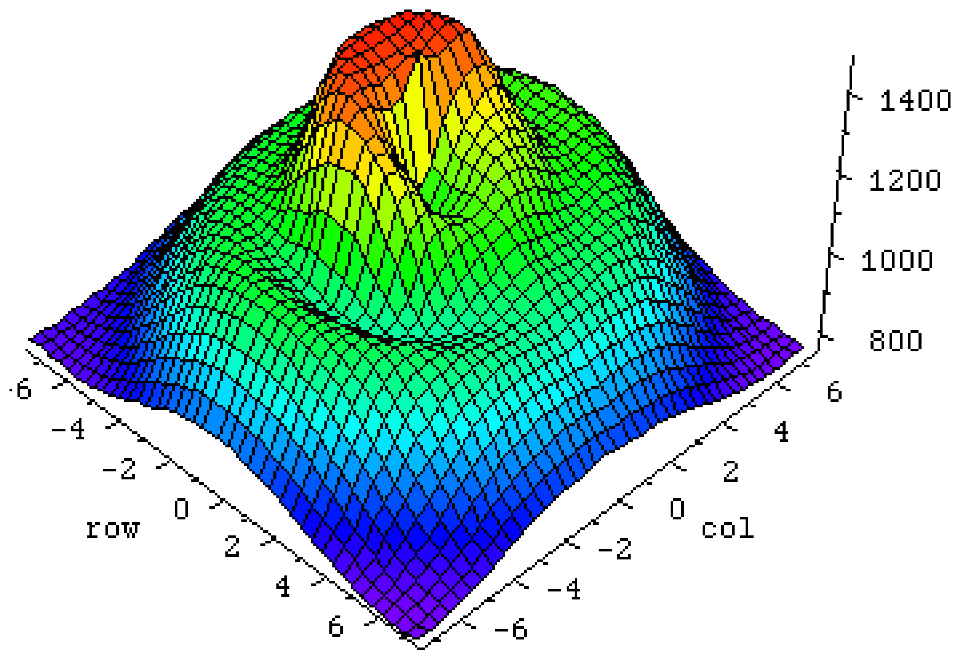


Figure 8.11: Reconstruction of an electron temperature profile for a discharge with MHD activity in TEXT-U for three phases of an MHD cycle.



e-m radiation detectors.

General considerations of a detector

Responsivity R_V

$$R_v = \frac{\text{rms output voltage}}{\text{rms power incident on detector}}$$

Detector time constant. Typical responsivity

$$R_v = \frac{R_0}{\sqrt{1+(\omega t)^2}}$$

R_V is a function of the modulation frequency of the incident radiation for cw operation. and of the pulse width for pulsed operation. Can be more than one time constant. (bolometers). Can be affected by temperature (germanium).

Types of detectors

Must change radiation into an electrical signal directly with electrons in a material. Since the electrons are bound to the lattice atoms or to impurity atoms a variety of interactions are possible: Photon effects, Thermal effects, Wave interaction effects. In the first case photons interact . See table

1. Internal effects	
1.1 Excitation of additional carriers	
1.1.1 Photoconductivity	
1.1.1a Electrically biased	
Intrinsic	Extrinsic
1.1.1b Microwave biased	
1.1.2 Photovoltaic effect	
1.1.2a $p-n$ junction	1.1.2b Avalanche
1.1.2c Schottky diode	1.1.2d $p-i-n$
1.1.2e Heterojunction	1.1.2f Bulk
1.1.3 Photoelectromagnetic effect	
1.1.4 Dember effect	
1.1.5 Phototransistor	
1.2 Free carrier interactions	
1.2.1 Photon drag	
1.2.2 Hot electron bolometer	
1.2.3 Putley detector	
1.3 Localized interactions	
1.3.1 Phosphor	
1.3.2 Infrared quantum detector	
1.3.3 Photographic film	
2. Photoemissive	
2.1 Photocathodes	
2.1.1 Negative electron affinity	2.1.2 Conventional
2.2 Gain mechanisms	
2.2.1 Gas avalanche	
2.2.2 Dynode multiplication	
2.2.3 Channel electron multiplication	

Thermal effects. A material absorbs the radiation, and the temperature increases.

Wave interaction effects: Interaction of e/m field with material results in internal property changes.

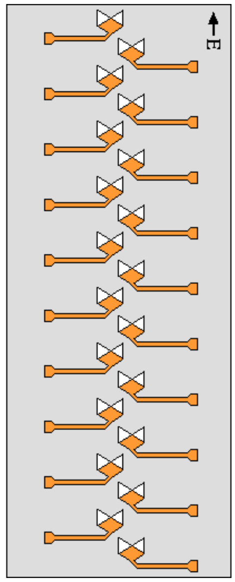
Photon effects: Basic distinction between photon and thermal: photon effects measure rate at which quanta are absorbed, whereas thermal effects measure rate of energy absorption. Photon detectors require incident radiation to have a certain minimum energy before they are detected, and are really selective of infra red. Thermal detectors respond to all the spectrum, but are much slower.

Example 1: An intrinsic 'photo diode' IR detector: radiation excites electrons near top of band gap of the valence band across the energy gap into states near the bottom of the conduction band, producing electron - hole pairs which in the case of a photo conductive sample changes the electrical conductivity. The semiconductor is designed to have an energy gap with width E_g related to the longest wavelength of the radiation to be detected. Mercury Cadmium Telluride (MCT) detectors, used for CO_2 laser wavelengths (10.4 mm) are photo conductive.

$$E_g = \frac{hc}{\lambda}; \quad E_g(eV) = \frac{1.24}{\lambda(\mu m)}$$

For CO_2 laser wavelengths need E_g = 0.116 eV. The material will respond to radiation of shorter wavelengths than this 'cutoff'. By mixing a semiconductor having a band gap < 0.1 eV with one having a gap > 0.1 eV one can tailor the material to that required. (Experiments show there is a continuous variation of the gap width with composition). So e.g. to get to 0.116 eV use CdTe with a wide band gap (E_g = 1.6 eV) with a semi metallic compound, HgTe, that can be thought of as a semiconductor with a negative energy gap of about 0.3 eV. The formula $Hg_{1-x}Cd_xTe$ represents the mixed crystal in which x denotes the mole fraction of CdTe. The gap varies linearly with x between the two end points, passing through zero at x = 0.15, and = 0.1 eV at x = 0.2 when temperature = 77^0K.

Example 2: Point contact and Schottky diode detectors. A rectifying action can occur at the interface between metal (tungsten, titanium) and a semiconductor crystal (e.g. GaAs). Incoming radiation is incident upon a whisker antenna. If polarization is parallel to the whisker, an electric field is created in the whisker which, in turn, is rectified at the junction. The rectified current is then detected as a voltage across a resistor. That the incoming radiation must have a polarization parallel to the whisker implies that the em radiation should be linearly polarized. Schottky diodes are strongly nonlinear, and mechanically stable. The stability is due to the notches which are provided on the crystal tip.



Bow Tie antennae

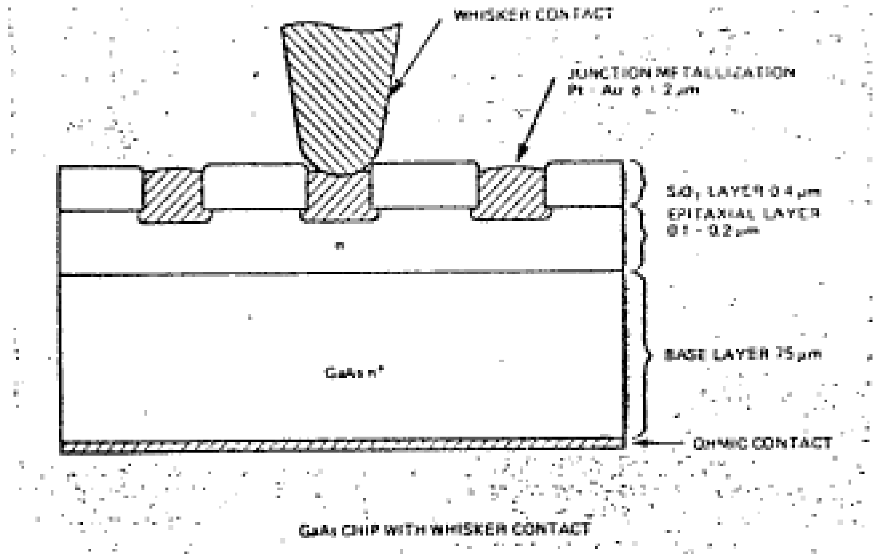


Fig.2.8. A cross-sectional view of Schottky barrier diode and whisker contact

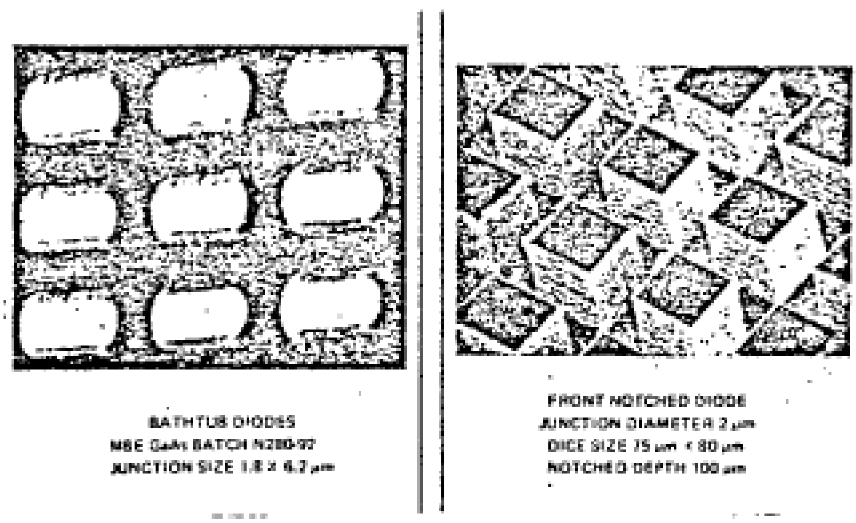


Fig.2.9. Two different array configuratios of the notches on a Schottky barrier diode .

TECHNIQUES FOR FABRICATING DEVICES AND CIRCUITS

EPITAXIAL GROWTH (MBE, VPE, LPE) EVAPORATION, SPUTTERING, ELECTROPLATING PLASMA DEPOSITION	MATERIAL GROWTH
DIFFUSION, ANNEALING, ALLOYING ION IMPLANTATION, PROTON BOMBARDMENT PLASMA OXIDATION, ANODIZATION	MATERIAL MODIFICATION
LITHOGRAPHY (UV, X-RAY, ELECTRON) ETCHING (CHEMICAL, ION, PLASMA) MACHINING, LASER CUTTING, SPARK EROSION	PATTERN GENERATION

Theory: Conduction is due to a combination of thermionic emmission and field emmission. Former: electrons have sufficient thermal energy to cross the barrier, the latter: electrons must tunnel across the gap. Relative proportions of each depend on temperature, doping density N of major carriers, and effective mass m* of majority carriers. High temperature increases proportion of electrons with sufficient energy to cross the barrier by thermionic effects. But image force lowering and narrowing of the potential barrier depends on N/m*

Initially barrier becomes thin enough that thermally excited carriers can tunnel through near the top of the band. This temperature dependent mode of current transport is called thermionic-field emmission. As impurity concentration is increased, barrier becomes so thin that a significant number of carriers can tunnel through even at the base of the band. This is field emission tunneling, and is temperature independent.

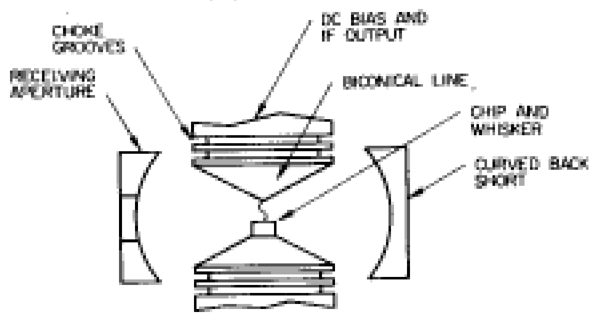
Write I-V characteristic as

$$I = I_s \exp\left[\frac{qV}{nkT}\right]$$

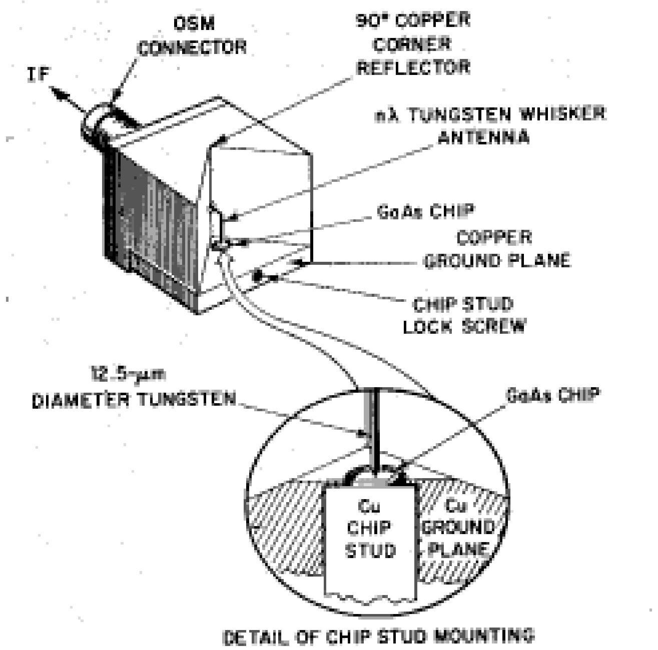
for I >> I_S. Thermionic emission is $n = 1$. Presence of a DC bias will lower the potential barrier and causes dominant conduction to change from thermionic to field emission at a specific I.

Construction

a) biconical mixer mounts. The Schottky diode chip and whisker are mounted at the apex of a biconical antenna. Radiation is guided through a circular coupling aperture to the antenna. The 1/4 wavelength grooves essentially open circuit the line at high frequencies, giving a high gain antenna. A back short is provided for resonating the structure. A teflon lens can used to focus the waves onto the contact point.



b) corner cube mixer mounts



90° CORNER REFLECTOR DIODE MOUNT

Here the configuration is that of a long wire antenna, with a corner cube to focus. The structure is equivalent to a four element phased array.

Heterodyne detection.

Originated in 1902 with Reginald Fessenden, awarded a patent 'relating to certain improvements... in systems were the signal is transmitted by waves differing in period and to the generation of beats by waves and the employment of suitable receiving apparatus responsive only to the combined action of waves corresponding in period to those waves'. heteros (Greek for other), dynamis (Greek for force). 1955 Forester observed mixing of two Zeeman components of a visible spectral line in a photo multiplier tube, in the first optical heterodyne experiment. 1966 Teich performed heterodyne with a CO_2 laser and a copper doped germanium photo detector at 4K

For heterodyne or coherent detection, the resultant intensity on the detector surface is not the same as video or incoherent detection. Incoherent case: resultant intensity is sum of individual intensities

$$I = \sum_i E_i^2$$

Coherent or heterodyne case: resultant is the square of the sum of coherent fields

$$I = \left(\sum_i E_i\right)^2$$

One field (local oscillator, LO) is dominant, while signal field is smaller. If local oscillator is derived from same source as the signal field then we have <u>homodyne detection</u>. Consider local oscillator E_{lo} and signal E_s at two frequencies. Total electric field at detector is

$$E = E_{lo} \cos(\omega_1 t) + E_s \cos(\omega_2 t)$$

Assume same polarization, then photo detector current or count rate is proportional to the square of the total electric field

$$i(t) = \frac{e\eta}{h\nu} P(t) \qquad\qquad P(t) = \frac{E^2(t) A}{z_0}$$

$i(t)$ is detector current, η is quantum efficiency, A is effective area, ν is frequency of incident photons, h is Planck's constant, z is impedance of medium surrounding detector. Then

$$i(t) = \frac{e\eta}{h\nu z_0} \begin{bmatrix} E_{l0}^2 \cos^2(\omega_1 t) + E_s^2 \cos^2(\omega_2 t) \\ + E_{l0} E_s \cos^2((\omega_1 - \omega_2)t) \\ + E_{l0} E_s \cos^2((\omega_1 + \omega_2)t) \end{bmatrix}$$

The detector cannot follow the instantaneous intensity at double frequency components if its resolution is larger than the period of the radiation. Post detector circuitry has a limited frequency response, so that only the averages of the first, second and third terms are observed. If $E_{lo} >> E_s$ then we only see the third term.

Note on 4th terms: quantum analysis shows if $h\nu >> kT$ then the double frequency does not appear.

Note on the important term: there are two ways that the system can evolve from its initial state to the final sate: by absorption of a photon from beam 1 (LO) or from beam 2 (signal). To distinguish from which bean we received the photon, its energy would have to be measured to within a value ΔE given by $\Delta E < h|\omega_1-\omega_2|$. From the uncertainty principle $\Delta E \Delta t > h$, so that the time required would be $\Delta t > h/\Delta E = |\omega_1-\omega_2|^{-1}$. The required measurement time is greater than the beat frequency (IF) and such a measurement would therefore wash out the time interference. Thus one cannot say from which beam the photon came.

Cerenkov radiation

Occurs when relativistic particles are present. These are de coupled from Coulomb collisions, but still interact with collective plasma oscillations. Cerenkov resonance condition is, with n = 0,

$$\omega_k - n\omega_{ce} = k_{\|}v_{\|}$$

Only direction along the B field matters, because gyrations average out others. The wave with frequency ω_k will grow unstable if the distribution function has a positive slope.

For negative n we have the anomalous Doppler effect. Longitudinal energy of electron is converted to transverse energy. Let electron emit a quantum of energy $\hbar\omega_k$ and parallel momentum $\hbar k_{\|}$. Then electron energy change is $\delta W = \delta W_{\|} + \delta W_{\pi} = -\hbar\omega_k$. Due to momentum conservation the longitudinal energy change is $\delta W = -v_{\|}\hbar k_{\|}$